Introduction To Circuit Analysis

The following definitions offer clarification for the student.

E=DC voltage. $\vec{\mathbf{E}}$ = vector voltage. v(t) = time varying voltage. I, $\vec{\mathbf{I}}$, and i(t) = current.

Fundamental Quantities

Characteristics that can be verified against base SI units, or derived quantities.

Coulomb (derived) (C) - One Ampere-second. The charge on approximately 6.24 X 10^{18} electrons. Charge on one electron is 1.6 X 10^{-19}C.

Voltage (derived) (V or E) - A volt is the Electromotive Force (EMF) required to energize a 1 watt load with 1 amp of current.

Resistance (derived) (R) - One ohm is the resistance needed to generate 1 Volt with 1 amp of current.

Current (base) (I) - This is a base standard of SI units, and is measured as the amount of current needed to create a force of 2 x 10^{-7} newtons per meter of length between two infinitely long, small, parallel wires 1 meter apart. It is also one Coulomb/second.

Capacitance (derived) (C) - One Farad is the size of a capacitor capable of holding 1 Coulomb at 1 Volt. C = Q / V

Inductance (derived) (L) - Inductance is the ratio of magnetic flux in a circuit to the current flowing in the circuit.

$$L = \varphi / I$$

One Henry provides 1 Volt when changing the current through a closed loop by 1 Amp/second.

Power (derived) (P) - A watt is defined as the amount of power that consumes one Joule per second. It is commonly defined as $\vec{\mathbf{P}} = \vec{\mathbf{E}}\vec{\mathbf{I}}$. Imaginary power does not convert to heat at the load. $\mathrm{Re}(\vec{\mathbf{P}})$ is always positive on passive circuits.

Energy (derived) (E) - A Joule is defined as the amount of energy dissipated by applying a force of one Newton (kg m/sec^2) over a distance of one meter. Or

$$J = \int P(t)dt$$

A kilowatt hour is 3.6 x 10^6 joules.

Frequency (derived) - Frequency (f) is the reciprocal of the period of the waveform (T). A period is the time interval after which the signal repeats.

F = 1/T. Radian frequency is given by

$$\omega = 2\pi f$$

Conventional Usage

Some terms or concepts with special meanings in Electronics.

Node - In a schematic or circuit, a point where multiple branches in the circuit join. A point where no voltage difference is possible.

Branch - In a schematic or circuit, a chain of components with a single current path.

Ground -

1) That portion of a circuit that can be tied to the earth or safety power connections without current being drawn. Or,

2) An arbitrary reference for a given circuit that cannot necessarily be equated with earth ground.

Load - That portion of a circuit that dissipates power or modifies input power, but does not generate power.

Source - That portion of a circuit capable of generating power.

Current Direction - Engineering convention is such that current flows from + to – terminals of a source. This is opposite to electron flow, but allows a number of other conventions to remain in force.

Open Circuit - A circuit through which no current flows.

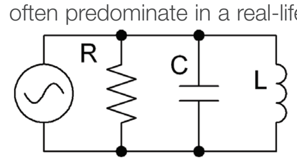
Short Circuit - A circuit across which no voltage can be developed.

Decibel (dB) -

Response in dB = $$10\log\left(\frac{P_{out}}{P_{in}}\right).$$

Often decibels are expressed in terms of voltages and currents. Because both voltage and current have a square relationship to power, expressing these quantities in dB requires multiplying the above equation by 2.

Simple Resonance - In AC circuits with parallel RLC elements, damping describes the amount of energy stored in a circuit compared to that consumed. In a purely resistive circuit, the energy stored is equal to zero. Reactive circuits (circuits containing capacitors and/or inductors) can store energy during a transient to be dissipated later. Caution: The analysis below does not include parasitics and non-optimum materials issues which often predominate in a real-life circuit.



Frequency Domain - If we define resonant frequency and quality factor (note that since at resonance

$$\omega_0 L = \frac{1}{\omega_0 C}$$

various shortcuts might be possible) then

Resonant Frequency -

$$\omega_0 = \sqrt{\frac{1}{LC}}$$

ω_0 is the resonant frequency in radians per second. It is defined by the inductance and capacitance without regard to resistance.

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Quality Factor -

$$Q = \frac{R}{\omega_0 L} \quad \text{or} \quad Q = \omega_0 RC$$

for parallel resonance. It is the inverse of this for series resonance.

Q defines the ratio of energy stored to energy dissipated during a cycle. Fundamentally, it gives a clue as to the amount of circulating energy in the resonant system that may not be visible outside the resonant system. Thus voltages across reactive components in series resonant circuits, and currents through reactive components in parallel resonant circuits can far exceed the voltages and currents that are visible across the resistive elements. Q is also the ratio of center frequency to the –3 dB bandwidth.

Bandwidth - The bandwidth of a reactive circuit is the difference between the two frequencies where response changes by 3 dB. Where DC is still full gain, it is the upper 3 dB point. Definition can be dependent upon circuit topology, and whether current or voltage is the measured parameter.

Time Domain - The time domain response of a circuit is calculated by writing the equation for the circuit, assuming some initial conditions and solving a differential equation.

$$\frac{d^2v}{dt^2} + \frac{dv}{RCdt} + \frac{1}{LC}v = 0$$

which assumes the final form to be

$$s^2 + \frac{s}{RC} + \frac{1}{LC} = 0$$

Use the quadratic equation to find the roots. Whether the discriminant shows two real roots, one root or two complex roots decides the damping.

Variables listed as A_n and φ are determined by initial conditions.

Under damped - $\frac{1}{2RC} < \omega_0$

An underdamped circuit generally rings and may take excessive time to approach the final value.

$$v(t) = A_1 e^{-\frac{\omega_0}{2Q}t} \sin(\omega_c t + \varphi)$$

where

$$\omega_c = \omega_0 \sqrt{\left(1 - \frac{1}{4Q^2}\right)}$$

Over damped -

$$\frac{1}{2RC} > \omega_0$$

An overdamped circuit exhibits no ringing, and takes excessive time to asymptotically approach the final value.

$$v(t) = A_1 e^{s_1 t} + A_2 e^{s_2 t}$$

where

$$s_1 = \omega_0 \left(\frac{-1}{2Q} + \sqrt{\left(\frac{1}{4Q^2} - 1\right)} \right) \quad \text{and}$$

$$s_2 = \omega_0 \left(\frac{-1}{2Q} - \sqrt{\left(\frac{1}{4Q^2} - 1\right)} \right)$$

Critically damped -

$$\frac{1}{2RC} = \omega_0$$

A critically damped circuit approaches its final value in the least possible time. This occurs when

$$v(t) = A_1 e^{st} + A_2 t e^{st}$$

where

$$s = \frac{-\omega_0}{2Q}$$

Passive Components

A basic description of passive components.

Ohms Law - $\vec{\mathbf{E}} = \vec{\mathbf{I}}\vec{\mathbf{Z}}$ where $\vec{\mathbf{Z}}$ is the vector sum of a resistive (positive x axis) component and combined inductive $(j\omega L)$ and capacitive $\left(\frac{1}{j\omega C}\right)$ components.

Since 1/j is –j, capacitance uses the negative y axis and inductance the positive y axis in the imaginary plane when summing impedance.

Resistance/Conductance -

Resistance is that property of a conductor that opposes the flow of current. It causes the dissipation of real power. The diagram here is used by covering the quantity to be identified.

What remains is the equation to calculate that quantity. E is calculated by multiplying I and R. I is calculated by putting E over R.

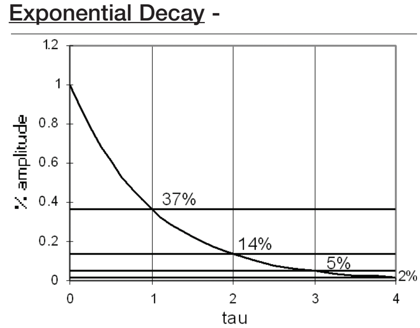
Capacitance - Capacitance is a measure of the propensity of parallel plates in a circuit to accumulate charge. No actual DC currents flow, but charges are added to and taken from the parallel plates giving the appearance of electron transfer as the voltage changes.

$$\mathrm{v(t)} = \frac{1}{C}\int \mathrm{i(t)}dt \quad \text{or} \quad i(t) = C\frac{dv(t)}{dt}$$

Inductance - Inductance is a measure of the unwillingness of a circuit or component to change current.

$$\mathrm{v(t)} = L\frac{d\,\mathrm{i(t)}}{dt}$$

Exponential Decay -



Normal RL or RC time constants are exponential decays. The above shows the remaining amplitude when t = time constant τ.

Digital Logic

Operators - The standard Boolean operators are described here. And, Or and Invert are fundamental. Nand, Nor, Xor and Xnor are derived. Nand and Nor are easy in hardware. Xor and Xnor are useful in arithmetic circuits.

And/Nand - The And function outputs a one whenever *all* inputs are one. Nand is the complement of And.

A	B	A * B	$\overline{A * B}$
0	0	0	1
0	1	0	1
1	0	0	1
1	1	1	0

Or - The Or function outputs a one whenever *any* input is one.

A	B	A + B	$\overline{A + B}$
0	0	0	1
0	1	1	0
1	0	1	0
1	1	1	0

Invert - The invert function outputs a one whenever its input is zero.

A	$\overline{A}$
0	1
1	0

Xor/Xnor (exclusive or/nor) - The Xor function outputs a one whenever one and *only* one input is one. Xnor is the complement of Xor.

A	B	$A \oplus B$	$\overline{A \oplus B}$
0	0	0	1
0	1	1	0
1	0	1	0
1	1	0	1

Postulates - The following are two of the primary Boolean algebra postulates. Care should be taken with semantics here. Boolean algebra, while similar to mathematic algebra in ways, is not identical in its definitions. For example, the Distributive property of Boolean algebra listed here doesn't match the distributive property of mathematical algebra. It is important to keep in mind that the "+" and "*"symbols have fundamentally different meanings in Boolean and mathematic algebra and therefore must be treated differently in some cases.

Commutative -

A+B = B+A

A*B = B*A

Associative -

(A+B)+C = A+(B+C)

(A*B)*C = A*(B*C)

Theorems -

The following are useful and provable from the postulates.

Distributive -

A*(B+C) = (A*B)+(A*C)

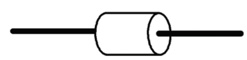
A+(B*C) = (A+B)*(A+C)

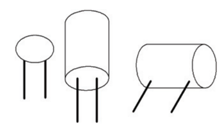
DeMorgan's Theorem -

$$\overline{(A + B)} = \overline{A} * \overline{B}$$

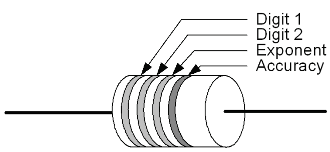
$$\overline{(A * B)} = \overline{A} + \overline{B}$$

Components

Axial Leaded - 

Radial Leaded - 

Color Codes



Black	0	Brown	1
Red	2	Orange	3
Yellow	4	Green	5
Blue	6	Purple	7
Grey	8	White	9

Accuracy: None, 20% Silver 10%, Gold 5%

Value = $(10*D1+D2)*10^{exponent}$

If exponent is Silver it is –2. If Gold –1.

1% and better resistors use four color bands for value.

Inductors and Capacitors sometimes use color bands or dots to indicate value.

Standard Resistor Values

Not every possible resistor value is practically available. These are the standard 5% values. 10% in bold. 20% in gray.

10	11	**12**	13	**15**	16	**18**	20
22	24	**27**	30	**33**	36	**39**	43
47	51	**56**	62	**68**	75	**82**	91

For information on precision resistors check

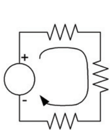
http://www.vishay.com/docs/31001/dectable.pdf

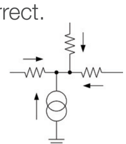
Inductors and capacitors generally use the same sequence. Often only the grayed box values, however.

Analysis Techniques

The following covers some of the standard analysis techniques, which depend on a few fundamental principles.

Kirchoff's Voltage Law - If you run around the loop and sum up all the voltages, the sum must be 0 for a steady-state condition. Depending which way you go, the source is positive and all the others are negative. If these are Z's rather than R's, then more care must be taken, but the result is still correct.





Kirchhoff's Current Law - Similarly, if you sum all the currents into (or out of) a node, the sum must be zero. Again, with non-resistive elements, care must be taken but the result must hold.

Nodal Analysis - Nodal analysis is the creation and solution of N-1 equations in N-1 unknowns for a circuit with N nodes. One node is defined as ground, so it doesn't need further solution. All the rest of the analysis is relative to that reference ground.

Procedure:

1) Write down the node voltages for those nodes on voltage sources. These are some of the equations.

($x = V_{n1}-V_{n2}$ where x is the value of the voltage source between N1 and N2)

2) Short the voltage sources out. This reduces the node count since two previously unconnected nodes are now tied together. Why does this work? Check source transformations.

3) Write the KCL equations for the remaining nodes, and express the currents as voltages across impedances. There should now be N-1 equations between those defined in 1 and 3.

4) Solve the equations. If the circuit includes Ls and Cs, this requires differential equations.

Mesh Analysis - Mesh analysis is the creation and solution of N-1 equations in N-1 unknowns. (Why N-1? It takes two nodes to build a mesh.)

Procedure:

1) Write down the meshes. Label them. Assure the mesh is two dimensional.

2) Write down the current sources.

3) Open the current sources. See Source Transformations for rationale.

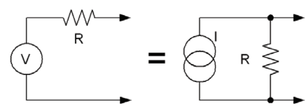
4) Write the equations for each remaining loop. Since there are only voltage sources and impedances left, the currents must be expressed in terms of volts and impedance.

5) Solve the resulting equations. Again, if there are Ls and Cs, this requires differential equations.

Source Transformations

There are two important reasons for more completely discussing sources. First, the analysis techniques suggest understanding them better. Second, you won't always have the source you need, and it is important to be able to comfortably swap to something you have.

In particular, current sources are much harder to make well than voltage sources.



Where V=IR or I=V/R.

This works due to the definitions of voltage and current sources. A voltage source maintains the same voltage regardless of the current. Thus the output impedance is 0 ($I_1R = I_2R$ requires R=0). Similarly, the output impedance of a current source is infinite (or undefined) since you can put any voltage across it and get the same current.

Superposition

In any linear circuit with multiple independent supplies, overall response can be calculated by calculating the response for each supply individually, with the remaining supplies shorted for voltage sources, or open for current sources.

Transforms

The following two transforms are critical to electronics calculation. The Laplace Transform provides a framework for filter design and frequency response calculations. The Fourier Transform allows you to examine time domain waveforms in the frequency domain. Primarily used to identify the transfer function of systems, these transforms are used somewhat interchangeably. The Fourier Transform has the advantage of being numerically calculable under certain circumstances.

Laplace Transform - The Laplace Transform creates a different data space, often known as the s-domain, to manipulate functions. These functions, in electronics, represent realizable circuits. The definition of the transform is:

$$Y(s) = \int_{t=0}^{\infty} e^{-st} y(t)dt$$

This transform makes dealing with integrals and derivatives quite simple, which is why it is popular for analyzing circuits containing inductors and capacitors.

Example:

$$Y(s) = \frac{1}{s+\alpha}$$

defines an exponential decay with the equation:

$$y(t) = e^{-\alpha t}$$

where α is the time constant of the waveform.

When transformed into the s-domain, circuits are expressed in terms of zero-terms/pole-terms. The roots of the various equations provide the location of the –3 dB corners for the elements. Non-real roots can cause oscillatory behavior.

The common transfer function H(s) is based upon Laplace Tranforms. Tables of common Laplace Transforms are available on-line or in most texts covering the subject.

S-domain equations are readily converted into z-domain equations, which are the basic descriptors of DSP operations.

Euler's Identity - Euler's Identity shows the relationship between polar and rectangular coordinates in the imaginary plane:

$$e^{j\theta} = \cos(\theta) + j\sin(\theta)$$

This identity is useful to keep in mind when interpreting the formulas defining Fourier Analysis.

Fourier Transform - The Fourier Transform is similar to the Laplace transform in operation. However, unlike the Laplace Transform, the Fourier Transform can be implemented for repetitive signals. The Fourier Transform proves that any waveform, no matter its shape, can be described as a sum of sinusoidal waveforms of various frequencies and magnitudes. Non-repetitive waveforms require an infinite sum of sinusoids over a continuous frequency band (hence the integral) to be described exactly. Repetitive waveforms require an infinite sum of sinusoids at discrete frequency intervals (hence the summation) to be exactly. While an infinite sum of sinusoids is required in either case to achieve mathematical perfection, it is generally the case that a limited set of the Fourier terms dominate the behavior of the waveform, and it is often true that some of the terms are equal to zero. It is therefore possible to make useful approximations by only considering the "relevant" Fourier terms. The finite bandwidth of a system guarantees that some possible terms will be irrelevant since they are outside the passband. The Fourier Transform is defined as:

$$\mathcal{F}(d(t)) = \int_{-\infty}^{\infty} d(t)e^{-j\omega t}dt$$

The Fourier Transform of an impulse provides constant amplitude stimulus at all frequencies. Thus you can look at the response and determine the transfer function.

A simplification for repetitive waveforms is of period 2π:

$$\mathcal{F}(x) = a_0 + \sum_{n=1}^{\infty} a_n \cos(nx) + \sum_{n=1}^{\infty} b_n \sin(nx)$$

where:

$$a_0 = \frac{1}{2\pi}\int_{-\pi}^{\pi} F(x)dx$$

$$a_n = \frac{1}{\pi}\int_{-\pi}^{\pi} F(x)\cos(nx)dx$$

$$b_n = \frac{1}{\pi}\int_{-\pi}^{\pi} F(x)\sin(nx)dx$$

This later equation basically states that a repetitive waveform can be represented as the sum of a number of sine functions. The sine and cosine terms effectively create a single function at a phase shift, so there can be many ways to represent this equation. Applying Euler's Identity to Fourier Transform expands it into a series of sine and cosine terms suggesting the commonality with the Fourier series.

H (ω) transfer functions are expected to be Fourier functions. Comparison of the definitions of H(s) and H (ω) suggest that they are very similar for all common electronic uses.

Frequency Response (Bode Plot)

The Bode plot takes the Laplace or Fourier based transfer function, and graphically shows expected magnitude and phase as a function of frequency. Zeros provide upwardly sloping lines with the inflection point at the -3 dB frequency. Poles, downwardly. These can be graphically overlayed on a dB vs log frequency plot and linear phase vs log frequency plot to provide a visualization of the overall transfer function. Line slope is 20 dB/decade * the exponent of the pole or zero. 20 dB/decade = 6 dB/octave.

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